# MAT1332, Winter 2008, Assignment #1 Solutions

1.

$$\int_{-2}^{2}(y^4+5y^3)dy = \left(\frac{1}{5}y^5+\frac{5}{4}y^4\right)|_{-2}^{2} = \left(\frac{32}{5}+\frac{80}{4}\right)-\left(\frac{-32}{5}+\frac{80}{4}\right) = \frac{64}{5} = 12.8$$

2.

$$\int_{-\pi/2}^{\pi/2}[x^2-20\sin(x)]dx = \left(\frac{1}{3}x^3+20\cos(x)\right)|_{-\pi/2}^{\pi/2} = \left[\frac{\pi^3}{24}+0\right]-\left[\frac{-\pi^3}{24}+0\right] = \frac{\pi^3}{12}$$

3. *First approach:* First find indefinite integral by substitution $u = 2\pi(x-2)$. Then $\frac{du}{dx} = 2\pi$ so $dx = \frac{du}{2\pi}$. Hence

$$\int \cos(2\pi(x-2))dx = \int \cos(u)\frac{du}{2\pi} = \frac{1}{2\pi}\sin(u)+C = \frac{1}{2\pi}\sin(2\pi(x-2))+C.$$

Then evaluate

$$\int_2^5 \cos(2\pi(x-2))dx = \frac{1}{2\pi}\sin(2\pi(x-2))\Big|_2^5 = \frac{1}{2\pi}(\sin(6\pi)-\sin(0)) = 0.$$

*Second approach:* Transform the limits of integration first. When $x=2$, $u = 2\pi(2-2) = 0$. When $x=5$, $u = 2\pi(5-2) = 6\pi$. Then the integral after substitution becomes

$$\int_2^5 \cos(2\pi(x-2))dx = \int_0^{6\pi}\cos(u)\frac{du}{2\pi} = \frac{1}{2\pi}\sin(u)\Big|_0^{6\pi} = \frac{1}{2\pi}\sin(6\pi)-\frac{1}{2\pi}\sin(0) = 0.$$

4. Using integration by parts, we have

$$\begin{array}{rclcrcl} u &=& x & \quad & v' &=& \cos(2x) \\ u' &=& 1 & & v &=& \frac{\sin(2x)}{2}. \end{array}$$

Thus

$$\begin{aligned}
\int_0^{\pi/2} x\cos(2x)dx &= \frac{x}{2}\sin(2x)\Big|_0^{\pi/2} - \int_0^{\pi/2}\frac{\sin(2x)}{2}dx \\
&= \left[\frac{x}{2}\sin(2x)+\frac{\cos(2x)}{4}\right]_0^{\pi/2} \\
&= \frac{\pi}{4}\sin(\pi)+\frac{\cos(\pi)}{4}-[0+\frac{\cos(0)}{4}] \\
&= 0-\frac{1}{4}-0-\frac{1}{4} \\
&= -\frac{1}{2}.
\end{aligned}$$

5. We use the substitution $u = 1 + 4t$. Then $\frac{du}{dt} = 4$, so $dt = \frac{du}{4}$. Thus

$$\int \frac{1}{1+4t} dt = \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |1 + 4t| + C.$$

(Don't forget to resubstitute.)

6. *First approach:* The length of a fish follows the differential equation

$$\frac{dL}{dt} = 6.48 e^{-0.09t}, \qquad L(0) = 5.$$

The solution to this pure-time differential equation is

$$L(t) = \frac{6.48}{-0.09} e^{-0.09t} + C = -72 e^{-0.09t} + C,$$

and the constant is obtained from $L(0) = -72 + C = 5$ so that $C = 77$. Hence,

$$L(0.5) = 77 - 72e^{-0.045} \approx 8.17, \qquad L(1.5) = 77 - 72e^{-0.135} \approx 14.09$$

The fish grows $L(1.5) - L(0.5) \approx 5.92$cm between ages 0.5 and 1.5.

*Second approach:* Using the Fundamental Theorem of Calculus, we don't need to evaluate the constant but can compute directly

$$\begin{aligned} L(1.5) - L(0.5) &= \int_{0.5}^{1.5} \frac{dL}{dt} dt \\ &= \int_{0.5}^{1.5} 6.48 e^{-0.09t} dt \\ &= -72 e^{-0.09t} \Big|_{0.5}^{1.5} \\ &= -72 e^{-0.135} - (-72 e^{-0.045}) \approx 5.92\text{cm}. \end{aligned}$$

7. *First approach:* The amount of chemical follows

$$\frac{dP}{dt} = 5e^{-2t}, \qquad P(0) = 2.$$

As above, we find $P(t) = -2.5e^{-2t} + C$ with $C = 4.5$. This gives $P(5) \approx 4.4999$ and $P(10) \approx 4.5$. Hence, the amount produced between times 5 and 10 is approximately 0.0001 moles.

*Second approach:* Using the Fundamental Theorem of Calculus, we don't need to evaluate the constant but can compute directly

$$\begin{aligned} P(10) - P(5) &= \int_{5}^{10} \frac{dP}{dt} dt \\ &= \int_{5}^{10} 5e^{-2t} dt \\ &= -2.5e^{-2t} \Big|_{5}^{10} \\ &= -2.5(e^{-20} - e^{-10}) \approx 0.0001\text{moles}. \end{aligned}$$